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A NOVEL METHOD OF SOLVING FUZZY PAYOFF MATRIX USING TRAPEZOIDAL FUZZY NUMBER

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Abstract
In this paper, an algorithm is used to solve the two person zero sum game whose imprecise values are represented by the fuzzy trapezoidal number. This problem is solved by using ranking method without converting the crisp value.

Keywords: Trapezoidal fuzzy number, Ranking of fuzzy trapezoidal number, Fuzzy matrix, Determinant of fuzzy matrix.

Introduction

Game theory problems occur frequently in Economics, Business Administration, Sociology, Political sciences, Military operations and so on. To challenge the uncertainty in Games, fuzzy set theory is an excellent source for studying the kind of Game in which the Payoffs are represented by fuzzy numbers. In 1965, fuzzy set were introduced by Zadeh L.A [2] to deal the problems while vagueness occurs. A fuzzy number is a quantity whose values are precise rather than exact as in the case with single valued numbers. Fuzzy numbers and their operations are the sources of fuzzy number theory which are used to modeling expert systems, cognitive computational models, measurement knowledge etc., In the model of game theory the real-life situations are converted to payoff matrix. Sometimes it is difficult to find the exact values of payoff. In such situation, the fuzzy payoff is the best choice for finding solution. [5] [6] given some properties on determinant and adjoint of square fuzzy matrix. Salim Rezvani, Mohammad Molani [8] studied the operations of trapezoidal fuzzy number. The value of the game for each player is assumed to be crisp numbers for the fuzzy payoff matrix. if the payoff matrix is fuzzy, then value of the game for player I and player II should also be fuzzy. This paper systematized as follows: Section 2 studied the definitions of trapezoidal fuzzy number, matrices and its arithmetic operations. The main part of this paper is to deals the mathematical formulation of fuzzy valued game problem and procedure for solving the same using matrix oddment method. Numerical examples are given.

Preliminaries

Fuzzy set [2]

A Fuzzy set $\tilde{A}$ in X (Set of real numbers) is a set of ordered pairs $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)/x \in X\rangle\}$ is called membership function of x in $\tilde{A}$ which maps X to [0,1].

α-cut of fuzzy set [2]

The α- cut of α- level set of fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. That is

A˜α = {⟨x/μ_A˜ (x) ≥ α⟩}

Support of fuzzy set [2]
The support of fuzzy set

A˜ is the set of all points x in X such that μA˜(x) > 0. That is

Support(A˜) = {x/μ_A˜ (x) ≥ 0}

Convex fuzzy set [2]
A fuzzy set A˜ is a convex fuzzy set if and only if each of its α - cut A˜α is a convex set.

Fuzzy number [2]
A fuzzy set membership function μA˜ : R → [0, 1] has the following characteristics A˜ is defined on the set of real numbers R is said to be a fuzzy number of its

A˜ is normal. It means that there exists an x ∈ R such that μA˜(x) = 1.
A˜ is convex. It means that for every x1, x2 ∈ R, μA˜[λx1+(1−λ)x2] ≥ min{μA˜(x1), μA˜(x2)}, λ ∈ [0, 1].
μA˜ is upper semi- continuous.
Supp(A˜) is bounded in R.

2.6 Triangular Fuzzy number[10]
It is a fuzzy number represented with three points as follows: A˜ = (a1, a2, a3).This

representation is interpreted as membership functions and holds the following conditions
 a1 to a2 is increasing function.
 a2 to a3 is decreasing function.
 a1 ≤ a2 ≤ a3.
It’s membership function is given by,

$$\mu_{\tilde{A}} = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ x > a_3 \end{cases}$$

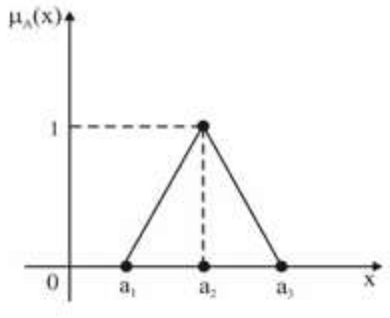

Figure 1: Triangular fuzzy number
Triangular fuzzy number

2.7 Trapezoidal Fuzzy number [3]
It is a fuzzy number represented with four points as follows: A˜ = (a1, a2, a3, a4).

This representation is interpreted as membership functions and holds the following conditions
a1 to a2 is increasing function.
a3 to a4 is decreasing function.
a1 ≤ a2 ≤ a3 ≤ a4.
It's membership function is given by,

$$\mu_{\tilde{A}} = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

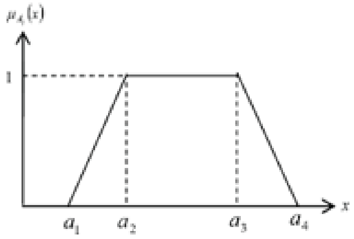

Figure 2: Trapezoidal Fuzzy number

2.8 α-cut of a trapezoidal fuzzy number [3]
An interval Aα shall be obtained as follows, for every α ∈[0, 1]. Thus Aα = [AL(α), AU (α)] = [(a2 − a1)α + a1, a4 − (a4 − a3)α].

2.9 Ranking of trapezoidal fuzzy number [3]
Let A˜ = [AL(α), AU (α)], 0 ≤ r ≤ 1 be a fuzzy number. The measure of A˜, M_0^Tra A˜ is calculated as follows

$$M_0^{Tra}\tilde{A} = \int_0^1 \propto (A_L(\propto), A_U(\propto) d \propto = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

2.10 Arithmetic operations on trapezoidal fuzzy number [9]
The following are the four operations that can be performed on trapezoidal fuzzy numbers: Let A˜ = (a1, a2, a3, a4) and B˜ = (b1, b2, b3, b4) then,
(i)Addition: A˜ + B˜ = (a1 + b1, a2 + b2, a3 + b3, a4 + b4).
(ii)Subtraction: A˜ − B˜ = (a1 − b4, a2 − b3, a3 − b2, a4 − b1) (iii)Multiplication: A˜.B˜ = (a, h, m, d) where a = min(a1b1, a1b4, a4b1, a4b4) h = min(a2b2, a2b3, a3b2, a3b3)
m = max(a2b2, a2b3, a3b2, a3b3)
d = max(a1b1, a1b4, a4b1, a4b4)
If AB < 0 then AB = (a4b4, a3b3, a2b2, a1b1)
If AB > 0 then AB = (a1b1, a2b2, a3b3, a4b4)
If A > 0 and B < 0 then AB = (a4b1, a3b2, a2b3, a1b4)
If A < 0 and B > 0 then AB = (a1b4, a2b3, a3b2, a4b1)

2.11 Determinant of trapezoidal fuzzy number [8]
The determinant of a square trapezoidal fuzzy matrix A˜ = (aij˜T zl) is denoted by (A) or det(A) and is defined as follows:

$$|A| = \sum_{h\in S_n} Signh \prod_{i=1}^{n} (a_i h(\tilde{\imath})^{Tzl})$$

$$= \sum_{h\in S_n} Signh \prod_{i=1}^{n} \left(a_i h(\tilde{1})^{Tzl}\right)\left(a_i h(\tilde{2})^{Tzl}\right) \dots (a_i h(\tilde{n})^{Tzl})$$

where (aih(˜i˜)T zl) are TrFN and Sn denotes the symmetric group of all permutations of the indices and Sign h = +1 or -1 according as the permutation h=$\begin{pmatrix} 1 & 2 & \dots & n \\ h(1) & h(2) & \dots & h(n) \end{pmatrix}$ is even or odd respectively.

The computation of det(A) involves several products of TrFNs. Since the product of two or more TrFNs is an approximate TrFN, the value of det(A) is also an approximate TrFN.

Remark: If A˜ = (a1, a2, a3, a4) is a trapezoidal fuzzy number then
(i) modulas of A˜ = mod(a1, a4) = mod(a2, a3).

3 Mathematical Formulation of Fuzzy Game Problem

Consider 2 competitors (called players) in the market. Let Player A has m Strategies A1, A2 . . . Am and player B has n strategies B1, B2 . . . Bn. Here it is assumed that each player A has his choices from amongst the pure strategies. Also it is assumed that player A is always gainer and player B is always looser. That is all pay offs are assumed in terms of player A. If player A chooses strategy Ai and player B chooses strategy Bj then pay off matrix to player A is

$$\text{Player B}$$

$$\textit{Player } A \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{pmatrix} \end{array}$$

3.1 Procedure for solving fuzzy game problem using matrix oddment method

Step 1: Let A = (aij) be a n × n fuzzy matrix. Obtain a new matrix C, whose first column is obtained from A by subtracting the successive column from its preceding column, till the last column of A is taken care of. Thus C is a n × (n − 1) matrix.

Step 2: Obtain a new matrix R from A, by subtracting its successive rows from the preceding ones,in exactly the same manner was done for the columns in step 1.Thus R is a (n − 1) × n matrix.

Step 3: Determine the magnitude of oddments corresponding to each row and each column of A. The oddment corresponding to ith row of A is defined as the determinant | Ci | where Ci is obtained from C by deleting ith row. Similarly oddment corresponding jth column of A = | Rj |, defined as determinant where Rj is obtained from R by deleting its jth column.

Step 4: Write the magnitude of oddments (after ignoring negative signs, if any) against their respective rows and columns.

Step 5: Check whether the sum of row oddments is equal to the sum of column oddments .If so, the oddments expressed as fractions of the grand total yields the optimum strategies. If not, the method fails.

Step 6: Calculate the expected value of the game corresponding to the optimum mixed strategy determined above for the row player (against any move of the column player).

4 Numerical Example

Consider the fuzzy payoff matrix with trapezoidal fuzzy numbers as elements

$$\begin{array}{lcc} & \textit{PlayerB} & \textit{RowMin} \\ \text{PlayerA} & \begin{pmatrix} (3,4,6,7) & (0,1,3,4) & (2,3,5,6) \\ (4,5,7,8) & (6,7,9,10) & (-1,0,2,3) \\ (5,6,8,9) & (-2,-1,1,2) & (1,2,4,5) \end{pmatrix} & \begin{array}{c} (0,1,3,4) \\ (-1,0,2,3) \\ (-2,-1,1,2) \end{array} \\ & & \\ \text{Columnmax} & \begin{array}{ccc} (5,6,8,9) & (6,7,9,10) & (2,3,5,6) \end{array} & \end{array}$$

Find the measure for each aij of the fuzzy game which is given in the following table for obtaining the minimum, maximum values.

Trapezoidal Fuzzy Number	Measures of TrFN M(T)= $\frac{a_1+2a_2+2a_3+a_{4_4}}{6}$
$\tilde{a}_{11}=$ (3,4,6,7)	$M_0^{Tray}\tilde{a}_{11} = 5$
$\tilde{a}_{12}=$ (0,1,3,4)	$M_0^{Tray}\tilde{a}_{12} = 2$
$\tilde{a}_{13}=$ (2,3,5,6)	$M_0^{Tra}\tilde{a}_{13} = 4$
$\tilde{a}_{21}=$ (4,5,7,8)	$M_0^{Tra}\tilde{a}_{21} = 6$
$\tilde{a}_{22}=$ (6,7,9,10)	$M_0^{Tray}\tilde{a}_{22} = 8$
$\tilde{a}_{23}=$ (-1,0,2,3)	$M_0^{Tra}\tilde{a}_{23} = 1$
$\tilde{a}_{31}=$ (5,6,8,9)	$M_0^{Tra}\tilde{a}_{31} = 7$
$\tilde{a}_{32}=$ (-2,-1,1,2)	$M_0^{Tray}\tilde{a}_{32} = 0$
$\tilde{a}_{33}=$ (1,2,4,5)	$M_0^{Tra}\tilde{a}_{33} = 3$

Maxmin =(0,1,3,4) and Minmax=(5,6,8,9). It has no saddle point; we use the oddment method to solve the problem.

Compute matrices C and R by subtracting the successive columns from the preceding columns and subtracting the successive rows from the preceding rows

$$C = \begin{pmatrix} (-7,-5,-1,1) & (-2,0,4,6) \\ (-2,0,4,6) & (-11,-9,-5,-3) \\ (-11,-9,-5,-3) & (-1,1,5,7) \end{pmatrix}$$

$$R = \begin{pmatrix} (-3,-1,3,5) & (2,4,8,10) & (-7,-5,-1,1) \\ (-3,-1,3,5) & (-12,-10,-6,-4) & (-2,0,4,6) \end{pmatrix}$$

To find the magnitude of oddments corresponding to each row and each column of A.

The oddment corresponding to 1st row of A is defined as | C1 |, where C1 is obtained from C by deleting 1st row. Similarly | R | is obtained from R.

$$|\, C_1 \,| = \begin{pmatrix} (-2,0,4,6) & (-11,-9,-5,-3) \\ (-11,-9,-5,-3) & (-1,1,5,7) \end{pmatrix} = (-79,-61,-25,-7)$$

$$|\, C_2 \,| = \begin{pmatrix} (-7,-5,-1,1) & (-2,0,4,6) \\ (-11,-9,-5,-3) & (-1,1,5,7) \end{pmatrix} = (-15,-5,15,25)$$

$$|\, C_3 \,| = \begin{pmatrix} (-7,-5,-1,1) & (-2,0,4,6) \\ (-2,0,4,6)) & (-11,-9,-5,-3) \end{pmatrix} = (-7,5,29,41)$$

$$|\, R_1 \,| = \begin{pmatrix} (2,4,8,10) & (-7,-5,-1,1) \\ (-12,-10,-6,-4) & (-2,0,4,6) \end{pmatrix} = (-24,-18,-6,0)$$

$$|\, R_2 \,| = \begin{pmatrix} (-3,-1,3,5) & (-7,-5,-1,1) \\ (-3,-1,3,5)) & (-2,0,4,6)) \end{pmatrix} = (9,7,3,1)$$

$$|\, R_3 \,| = \begin{pmatrix} (-3,-1,3,5) & (2,4,8,10) \\ (-3,-1,3,5)) & (-12,-10,-6,-4) \end{pmatrix} = (-14,-14,-14,-14)$$

Here

$$|R_1| = (-24, -18, -6, 0) \qquad |C_1| = (-79, -61, -25, -7)$$

$$|R_2| = (9, 7, 3, 1) \qquad |C_2| = (-15, -5, 15, 25)$$

$$|R_3| = (-14, -14, -14, -14) \qquad |C_3| = (-7, 5, 29, 41)$$

where mod(a1,a3) = mod(a2, a4).

Therefore the augmented payoff matrix is , Player B Row Oddments

$$
\begin{array}{llll}
 & \textit{Player B} & & \textit{Row Oddments} \\
\textit{Player A} & \begin{pmatrix} (3,4,6,7) & (0,1,3,4) & (2,3,5,6) \\ (4,5,7,8) & (6,7,9,10) & (-1,0,2,3) \\ (5,6,8,9) & (-2,-1,1,2) & (1,2,4,5) \end{pmatrix} & & \begin{array}{c} (0,1,3,4) \\ (-1,0,2,3) \\ (-2,-1,1,2) \end{array} \\
\textit{Column oddments} & (-24,-18,-6,0) \quad (9,7,3,1) \quad (-14,-14,-14,-14) & &
\end{array}
$$

$$\text{Sum of the row oddments} = -24+0+9+1-14-14=-42$$

$$\text{Sum of the column oddments} = -79-7-15+25-7+41=-42$$

$$\text{Strategy for Player A} \quad =\left(\frac{43}{42}\frac{-5}{42}\frac{-17}{42}\right)$$

$$\text{Strategy for Player B} = \left(\frac{12}{42}\frac{-5}{42}\frac{14}{42}\right)$$

$$\text{Value of the game} = \frac{43}{42}(3,4,6,7) + \frac{-5}{42}(4,5,7,8) + \frac{-17}{42}(5,6,8,9)$$

$$\vartheta = \left(\frac{24}{42}\frac{45}{42}\frac{87}{42}\frac{108}{42}\right)$$

The obtained value of the game is also a trapezoidal fuzzy number. Therefore, crisp value of the solution of fuzzy valued game problem without converting to crisp valued problem using measure = 3.14 = 1.57.

Conclusion

In this paper we have considered 3 × 3 fuzzy payoff matrix whose elements are trapezoidal fuzzy numbers. We proved that the optimal solution of the fuzzy valued game problem of trapezoidal fuzzy number using matrix oddment method is also a trapezoidal fuzzy number.

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